Math Review – Part B

Intermediate Level

(Up to end of MAT 034)

A scientific calculator is allowed.

Answers provided in final section.

Updated July 2017

Intermediate Level Algebra

ORDER OF OPERATIONS

Remember the acronym **BEDMAS** for the order of operations. The letters stand for:

Brackets or parenthesis

Exponents

Division
Multiplication (left to right)

Addition
Subtraction (left to right)

This is an internationally agreed-upon system for dealing with multiple operations.

Examples:

Calculate: $2 - 3(3 - 5)$

<u>Solution</u>	$2 - 3(3 - 5)$
Step 1: subtract $3-5$ within the brackets	$= 2 - 3(-2)$
Step 2: multiply $3(-2)$	$= 2 - (-6)$
Step 3: subtract $2 - (-6)$	$= 2 + 6$
Step 4: add $2 + 6$	$= 8$

Calculate: $8^2 - 3^3 \times 10 \div (-2)$

<u>Solution</u>	$8^2 - 3^3 \times 10 \div (-2)$
Step 1: calculate 8^2 and 3^3 (exponents)	$= 64 - 27 \times 10 \div (-2)$
Step 2: multiply 27×10	$= 64 - 270 \div (-2)$
Step 3: divide $-270 \div (-2)$	$= 64 + 135$
Step 4: add $64 + 135$	$= 199$

Calculate: $2(3+4)-5\times\frac{1}{2}$

<u>Solution</u> $2(3+4)-5\times\frac{1}{2}$

Step 1: add $(3+4)$ in brackets $=2(7)-5\times\frac{1}{2}$

Step 2: multiply $2(7)$ $=14-5\times\frac{1}{2}$

Step 3: multiply $5(\frac{1}{2})$ $=14-5\times\frac{1}{2}$

Step 4: subtract $14-\frac{5}{2}$ $=11\frac{1}{2}$

Calculate the following.

1. $9+2\times 8=$

2. $9\times 8+7\times 6=$

3. $4\times 5^2=$

4. $(12-8)-4=$

5. $15(4+2)=$

6. $8\times 2-(12-0)\div 3-(5-2)=$

7. $\frac{80-6^2}{9^2+3^2}=$

8. $32-8\div 4-2=$

9. $95-2^3\times 5\div (24-4)=$

10. $1000\div 100\div 10=$

Signed Numbers

REVIEW

Many situations in the world are represented by negative quantities. For example, temperatures below zero, an overdrawn bank account, etc. Mathematically we use signed numbers i.e. positive and negative numbers. Addition, subtraction, multiplication, and division can be carried out using signed numbers. A summary of the rules follows:

Rules for Adding and Subtracting Signed Numbers

The sum of positive numbers is positive.

$6+7=13$

The sum of negative numbers is negative.

$(-2)+(-3)=-5$

When adding positive and negative numbers, find the difference and take the sign of the larger.

$(-4)+3=-1$

When subtracting positive and negative numbers, change the sign of the number being subtracted and proceed as in addition. This is also referred to as “adding the opposite.”

$5-(-2)=5+2=7$

Rules for Multiplying Signed Numbers

$+\times+=+$	positive $\times$ positive = positive
$-\times-=+$	negative $\times$ negative = positive
$-\times+=-$	negative $\times$ positive = negative

Examples

$5\times(-3)\times(-2)=30$

$(-1)\times(-2)\times(-3)\times(-4)=24$

$(-2)\times(-3)\times(-4)=-24$

Rules for Dividing Signed Numbers

$+ \div + = +$	positive ÷ positive = positive
$- \div - = +$	negative ÷ negative = positive
$- \div + = -$	negative ÷ positive = negative
$+ \div - = -$	positive ÷ negative = negative

Examples:

$8 \div 4 = 2$ $\quad$ $(-6) \div (-3) = 2$

$(-12) \div 3 = -4$ $\quad$ $8 \div (-2) = -4$

Solve the following.

11. $5 + (-2) =$

12. $(-3) - (-4) =$

13. $(-2) + (-8) =$

14. $(-5) \times 2 =$

15. $(-8) \times (-4) =$

16. $16 \div (-4) =$

17. $(-24) \div (-3) =$

18. $(-3) + (-2) + (-4) =$

19. $2 \times (-3) + 5 \times (-4) - 5 =$

20. $(-15) \div (-3) =$

Ratio and Proportion

(Go back to the fundamental section to review ratio and proportion.)

Reduce these ratios to lowest terms.

21. $24:60$

22. $18:27:42$

Determine whether the following ratios are equal.

23. $1:6$ and $36:216$

24. $2\frac{1}{2}:\frac{3}{4}$ and $20:6$

Solve the following proportions for the given variable.

25. $a:\frac{1}{6}=14:2$

26. $4\frac{1}{4}:8=d:12$

27. In a poll of 3000 children, 700 children said they preferred chocolate milk and the rest said they preferred regular milk. If you were offering milk for 5000 children, how many would you expect to want to drink regular milk?

28. In Courtenay, a bill of 575 kilowatt hours of electricity was $38.81. In Comox, a bill of 831 kwh is $58.10. In which community is electricity cheaper? (These numbers are not based on fact.)

Percent

(Go back to fundamental section to review percent.)

29. Convert to decimals.

    a) 66%

    b) 235.7%

30. Convert to fractions.

    a) 255%

    b) $66\frac{2}{3}\%$

31. Convert to percentage.

    a) 0.00006

    b) $6\frac{3}{4}$

32. Solve the following.

    a) In a candy dish, there are three types of chocolates. Twelve are white chocolate, 18 are dark chocolate, and 6 are milk chocolate. What percentage of the chocolates are dark chocolate?

33. Find the simple interest earned on $20,000 for 4 years at 8%. Use the formula $I = Prt$

Measurement

(Go back to fundamental section to review measurement.)

Area Conversions

If you are converting from km to m you move the decimal 3 places to the right.

For example: 1 km = 1000 m

BUT if you are converting from km^2 to m^2 you move the decimal 3 places to the right TWICE.

For example: 1 km^2 = 1 000 000 m^2

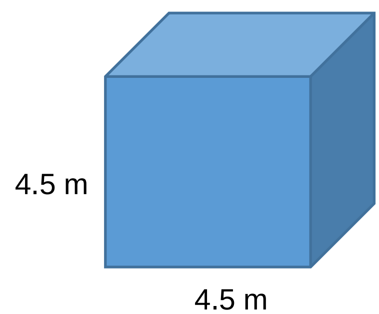
Convert the following.

34. $24\,\text{hm}^2 =$ ____________ m^2

35. $4.32\,\text{km}^2 =$ ____________ m^2

36. $0.00215\,\text{m}^2 =$ ____________ cm^2

37. Find the area of one wall of this square cube.



Volume / Mass Conversions

If you are converting from km to m you move the decimal 3 places to the right.

For example: 1 km = 1000 m

BUT if you are converting from km^3 to m^3 you move the decimal 3 places to the right THREE TIMES.

For example: 1 km^3 = 1 000 000 000 m^3

Convert the following.

38. $25.5\,\text{km}^3 =$ ____________ m^3

39. $456.78\ \text{cm}^3 =$ ____________ m^3

40. $0.0077\,\text{dal}^3 =$ ____________ dl^3

Convert the following. (See Appendix A & B on page 52 and 53 for the conversion charts.)

41. $64\,\text{ml} =$ ____________ cm^3

42. $46,277\,\text{L} =$ ____________ m^3

43. $0.054\,\text{t} =$ ____________ kg

44. $1\,\text{t} =$ ____________ g

45. Noreen is 5 feet 6 inches. Find her height in centimeters.

46. The 26.2 miles marathon is_______________ km?

47. My new truck weights 3560 lbs. What is the mass of the truck in kg?

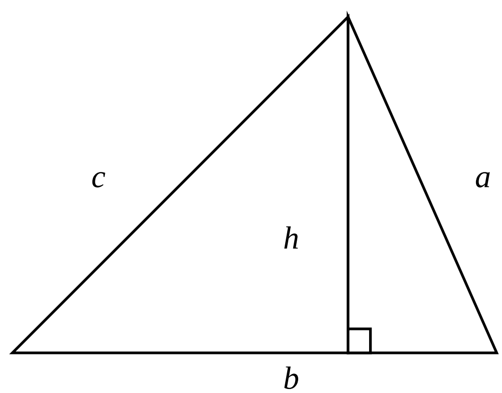
48. How many liters of water will fit into a container that measures 50 cm by 50 cm by 50 cm?

Perimeter, Area, and Volume

REVIEW OF TRIANGLES

A triangle is a closed geometric figure with three sides, in which each side is a straight line segment.

Let a, b, and c be the lengths of the sides of a triangle and h be the height (the distance from one side to the opposite vertex).



The perimeter of a triangle is the sum of the lengths of the sides.

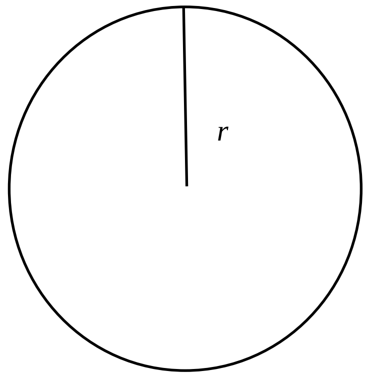
$$P = a + b + c$$

The area of a triangle is one-half the base times the height.

$$A = \frac{1}{2} \times 4 \times 2 = 4$$

REVIEW OF CIRCLES

Let $r = \text{radius}$, $C = \text{circumference}$, $d = \text{diameter}$, $A = \text{area}$, $\pi = 3.14$



The **radius** of a circle is the distance r from the centre to any point on the circle. The **diameter** is the distance d across the circle or twice the radius.

$$d = 2r$$

The **circumference** (distance around) of a circle is π times the diameter.

$$C = \pi d = 2\pi r$$

The **area** of a circle is the square of the radius (r^2) times π.

$$A = \pi r^2$$

Example:

A circle with radius $= 5$ has the following diameter, circumference, and area:

$$d = 2r \qquad d = 2 \times 5 = 10$$

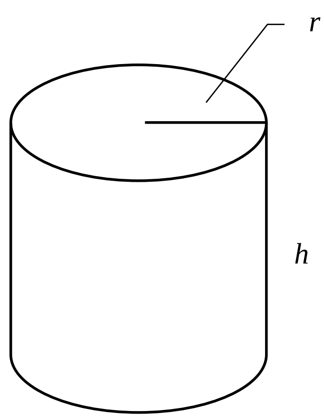
$$C = \pi d \qquad C = \pi 10 = 31.416$$

$$A = \pi r^2 \qquad A = \pi \times 5^2 = 25\pi = 78.54$$

REVIEW OF VOLUME

The volume of a cylinder is the height times the area of the base.

$$V = \pi r^2 h$$



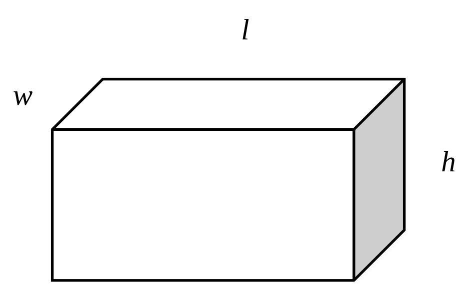
Example:

If a cylinder has a height $= 10$ and radius $= 5$ then the volume is

$$V = \pi(5)^2 \times 10 = 250\pi = 250 \times 3.14 = 785$$

The volume of a box is the length times the width times the height.

$$V = l \bullet w \bullet h$$



Example:

If a box has length $= 10$, width $= 6$, and height $= 5$, then the volume is

$$V = 10 \times 6 \times 5 = 300$$

Volume is expressed in cubic measurements, cm^3, m^3, km^3, etc.

Problems involving perimeter, area, and volume.

49. George is making a rectangular patio that measures 6 m by 8.5 m and is .25 m deep. How many cubic metres (m^3) concrete does he need?

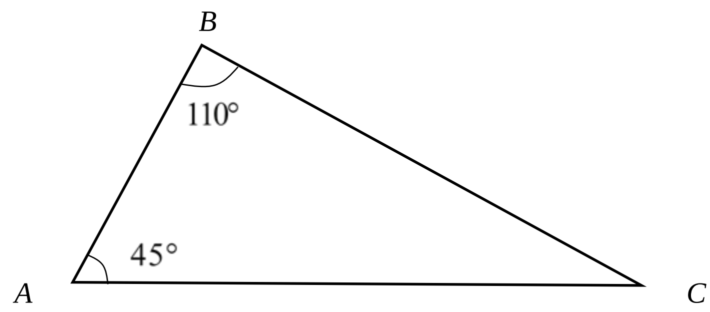
50. What is the diameter of a circle that has an area of 706.5 square cm (cm^2)?

51. If the circle in Question #50 formed the base of a cylinder 16 cm high, what would be the volume of the cylinder? V of cylinder is $V = \pi r^2 h$

Intermediate Level Geometry

TRIANGLES

The sum of the three angles of any triangle is 180°. Knowing this you can find the size of a missing angle.



Example:

$\angle ABC = 110^\circ$

$\angle BAC = 45^\circ$

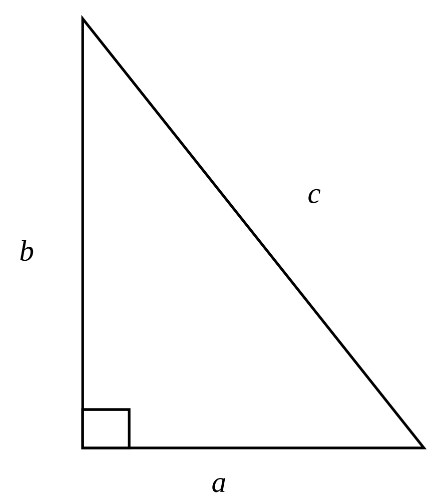
Then:

$$\begin{aligned} 110^\circ + 45^\circ + \angle BCA &= 180^\circ \\ 155^\circ + \angle BCA &= 180^\circ \\ \angle BCA &= 180^\circ \\ \angle BCA &= 180^\circ - 155^\circ \\ \angle BCA &= 25^\circ \end{aligned}$$

PYTHAGOREAN THEOREM

In a right angled triangle the square of the hypotenuse equals the sum of the square of the other two sides. The hypotenuse is the side opposite the 90° angle.

Example:



$c^2 = a^2 + b^2$

$a = 4$

$b = 7$

Find the length of side c

$c^2 = 7^2 + 4^2$

$c^2 = 49 + 16$

$c^2 = \sqrt{65}$

$c = 8.06$

52. How long a ladder would be needed to reach a window that is 4 m off the ground, if the base of the ladder is 3 m from the wall? (Use Pythagorean theorem.)

END OF MATH 033

Polynomials

A polynomial is an algebraic expression made up of terms that are added or subtracted together. The polynomial $2x^2 - 3x + 6$ is made up of the terms $2x^2$, $-3x$, 6.

The coefficient of a term is the number in front of the variable. In the term $-9x$ the coefficient is -9 and the variable is x.

Polynomials can be evaluated when each variable is given a numerical value.

Example 1:

Evaluate $2x^3y + x$ when $x = 2$ and $y = -5$

Solution:

Replace x with 2 and $y = -5$

$$\begin{aligned} 2x^3y + x &= 2(2)^3(-5) + 2 \\ &= 2(8)(-5) + 2 \\ &= 16(-5) + 2 \\ &= -80 + 2 \\ &= -78 \end{aligned}$$

Example 2:

Evaluate $\frac{t^2 - t}{3}$ when $t = -3$

Solution:

Replace t with -3

$$\begin{aligned} \frac{t^2 - t}{3} &= \frac{(-3)^2 - (-3)}{3} \\ &= 9 + \frac{3}{3} \\ &= 4 \end{aligned}$$

Evaluate the following.

Evaluate each polynomial when $x = -2$

53. $-5x + 6$

54. $3x^2 - 2x + 1$

Adding Polynomials

Polynomials can be added or subtracted if their terms are “like terms.” The polynomial is said to have been simplified.

e.g. Simplify the polynomial by combining like terms.

Add $6x^3 + 7x^2 + 3$ and $-2x^3 - 4x^2 - 5$

Rearrange the terms so the x^3 terms and the x^2 terms and the numbers are together.

$6x^3, -2x^3, +7x^2, -4x^2, +3, -5$

Now combine (add or subtract) the like terms.

$4x^3 + 3x^2 - 2$

Simplify the following by combining like terms.

55. $-5x^3 + 3x^2 + 8$ and $-2x^3 - 4x^2 - 5$

56. $6a - 3b + 12c - 10b - 8c + 2$

57. $ab^2 - 3ab + 4ab^2 - 10ab - 8c$

Subtracting Polynomials

Subtracting polynomials can be thought of in two different ways.

Method 1

When there is a negative sign in front of a bracket and when the brackets are removed the terms inside the brackets are written as their opposites.

eg.

$$\begin{aligned} 9x-(2x+5) &= 9x-2x-5 \\ &= 7x-5 \end{aligned}$$

Method 2

There is a law in math called the **Distributive Law**. It says the negative sign in front of the bracket is really an understood -1.

$$9x-1(2x+5)$$

To remove the bracket you must multiply the -1 to the $2x$ to make $-2x$ and the -1 to the 5 to make -5.

$$\begin{aligned} 9x-1(2x+5) &= 9x-2x-5 \\ &= 7x-5 \end{aligned}$$

Subtract. Remove brackets and combine like terms.

58. $x^2-2x+5-(3x^2-2)$

59. $-(6x^4+3x^3-1)-(3x^2+x+1)$

60. $5a-(2a^2+a)$

Multiplying Polynomials

Monomial by monomial

1. Multiply the coefficient of each term
2. Multiply the variables and add the exponents of any like variables.

eg.

$$a^m \times a^n = a^{m+n}$$

$$a^3 \times a^5 = a^{3+5} = a^8$$

$$2x(x^2 - 5x + 4) = 2x^3 - 10x^2 + 8x$$

Binomial by binomial

1. When multiplying binomials together use the **FOIL** method.

First terms	in attached bracket
Outside terms	in each bracket
Inner terms	in each bracket
Last terms	in each bracket

eg.

$$\begin{aligned}(x+3)(x+5) &= x^2 + 5x + 3x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

Polynomial by polynomial

Multiply $3x^2 - 7x + 4$ by $(-2x - 5)$

You need to recognize that there will be 6 terms after multiplying.

$$-6x^3 - 15x^2 + 14x^2 + 35x - 8x - 20$$

Now collect like terms.

$$-6x^3 - x^2 + 27x - 20$$

Multiply the following.

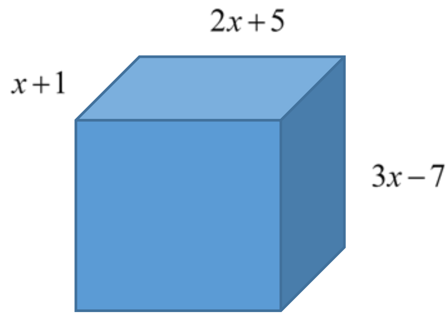
61. $-3a(a^2 - a + 4)$

62. $(2x - 3)(2x - 6)$

63. $(x^2+6)(2x^2-4x+5)$

64. $(3x-2)(4x+1)(2x+3)$

65. Find the volume of the figure below.



Special Products

a) Sum and Difference of Two Terms

$(A+B)^2 = A^2 + 2AB + B^2$
$(A-B)^2 = A^2 - 2AB + B^2$

eg. $(m+3)^2 = m^2 + 6m + 9$

eg. $(6x-3)^2 = 36x^2 - 36x + 9$

Multiply the Following.

66. $(2x+5)^2$

67. $(3x-1)^2$

68. $(6-2a)^2$

Dividing Polynomials

Polynomial by monomial

Divide each term of the polynomial by the monomial.

eg. $\dfrac{a^m}{a^n} = a^{m-n},\ a \neq 0$

$\dfrac{4^4}{4^2} = 4^{4-2} = 4^2 = 16$

$\dfrac{6x^2 + 4x}{2x} = \dfrac{6x^2}{2x} + \dfrac{4x}{2x} = 3x + 2$

Divide the following.

69. $-2p^3 + p^2 - 3p$ by p

70. $x^2y^2 + xy^3$ by xy^2

71. $\pi r^2 - 3\pi r$ by πr

72. Use the formula $t = \frac{d}{r}$ to find the time (t) it takes a car to travel a distance of 225 km at a rate of speed of 80 km/h.

73. A rental company charges a rate of $90.00 a week to rent a car (fixed value) then a charge of $0.06 per kilometer (depends on distance). If you drive 800 km, how much does it cost to rent the car for a week?

The equation is: Fixed fee plus cost times number of kilometers equals total cost.

Factoring

Factoring means to write a number as the multiplication of two or more numbers.

eg. The factors of 12 are:

$$12 = 1 \times 12$$
$$12 = 2 \times 6$$
$$12 = 3 \times 4$$

The PRIME factors of 12 are:

$$12 = 2 \times 2 \times 3$$

eg. You can reduce factors to lowest terms by factoring and then reducing by cancelling.

$$\frac{36}{40} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 5} = \frac{9}{10}$$

eg. You can find the greatest common factor by listing all the factors and then finding the largest one.

Factors of 20 are: 1, 2, 4, 5, 10, 20

Factors of 30 are: 1, 2, 3, 4, 6, 10, 15, 30

The **Greatest** common factor between 20 and 30 is **10**.

eg. You can find the greatest common factor for numbers and letters by listing all the factors and then finding the largest one.

Factors of $8y^3$ are: 1, 2, 4, 8, y, y, y

Factors of $20y^2$ are: 1, 3, 4, 5, 10, 20, y, y

The **Greatest** common factor between $8y^3$ and $20y^2$ is $4y^2$.

eg. To factor a polynomial of two or more terms, remove the greatest common factor from all terms including both numbers and variables.

eg.
$$\begin{aligned} 8m^2 + m &= 8mm + m \\ &= m(8m + 1) \end{aligned}$$

eg.
$$\begin{aligned} 6b^2 - 2b + 10b^3 &= 2 \bullet 3bb - 2b + 2 \bullet 5bbb \\ &= 2b(3b - 1 + 5b^2) \end{aligned}$$

Factor the following.

74. $8x+12y$

75. $2x^3y^3-8x^2y^2$

76. $8x^3-20y+12y-16$

Factoring Simple Trinomials

eg. $y^2+12y+20$

Step 1: Make two brackets ( ) ( )

Step 2: Split the first term $(y \quad)(y \quad)$

Step 3: Now ask yourself, "What two numbers multiply to 20 (the last term) but add to the coefficient +12 from the middle term?"

Step 4: The factors 2 and 10 add to 12

Step 5: The answer is $(y + 2)(y + 10)$

eg. $5y^2-5y-30$ ****Take out a common factor first $5(y^2-y-6)$

Step 1: Make two brackets 5( ) ( )

Step 2: Split the first term $5(y \quad)(y \quad)$

Step 3: Now ask yourself, "What two numbers multiply to -6 (the last term) but add to the coefficient -1 from the middle term?"

Step 4: The factors -3 and 2 add to -1

Step 5: The answer is $5(y+2)(y-3)$

Factor the following.

77. x^2-9x+8

78. $r^2-6r-16$

79. y^2+y-42

Factoring a Difference of Squares

$$A^2 - B^2 = (A+B)(A-B)$$

Factor $x^2 - 4$

Step 1:	Make two brackets	$(\quad)(\quad)$
Step 2:	Split the first term	$(x\quad)(x\quad)$
Step 3:	Split the second term	$(x\quad 2)(x\quad 2)$
Step 4:	Now, put a plus (+) sign in one bracket and a minus (–) sign in the other bracket. The order of the brackets does not matter.	
Step 5:	The answer is $(x+2)(x-2)$	

Factor $9m^2 - 49$

Step 1:	Make two brackets	$(\quad)(\quad)$
Step 2:	Split the first term	$(3m\quad)(3m\quad)$
Step 3:	Split the second term	$(3m\quad 7)(3m\quad 7)$
Step 4:	Now, put a plus (+) sign in one bracket and a minus (–) sign in the other bracket. The order of the brackets does not matter.	
Step 5:	The answer is $(3m-7)(3m+7)$	

Factor the following.

80. $x^2 - 16$

81. $2x^2 - 50$

82. $9x^2 - 4$

Word problems.

83. If a truck can travel at 120 km/h for 4 hours, how far can it go?

84. If a horse can run 20 km in 5 hours, how fast can it run?

Solving Equations

Most simple equations can be solved by using a simple approach:

What you do to one side of an equation you must do to the other.

Remember these simple guidelines:

a. Clear fractions by multiplying by the denominator.
b. You can add or subtract a quantity from both sides of the equation.
c. You can divide every term by whatever is in front or behind what you are solving for.

Examples:

Solve $x+4=12$ $\quad$ *subtract 4 from both sides*

$$x=12-4$$
$$x=8$$

Solve $2x+3=15$ $\quad$ *subtract 3 from both sides*

$$2x=15-3$$
$$2x=12$$
$$x=\frac{12}{2} \qquad \textit{divide both sides by } 2$$
$$x=6$$

Solve $\frac{3}{4}x+5=8$ $\quad$ *clear the fraction by multiplying all terms by 4*

$$(4)\frac{3}{4}x+(4)5=(4)8$$
$$3x+20=32 \qquad \textit{subtract } 20 \textit{ from both sides}$$
$$3x=32-20$$
$$\frac{3}{3}x=\frac{12}{3} \qquad \textit{divide both sides by } 3$$
$$x=4$$

Equations Involving Negative Numbers

Solve $3x-7=-13$ *add 7 to both sides*

$$3x=-13+7$$

$$3x=-6 \quad \textit{divide both sides by } 3$$

$$x=\frac{-6}{3}$$

$$x=-2$$

Solve $4(x+6)=16$ *distribute 4 to both terms in the brackets*

$$4x+24=16 \quad \textit{subtract } 24 \textit{ from both sides}$$

$$4x=16-24$$

$$4x=-8 \quad \textit{divide both sides by } 4$$

$$\frac{4x}{4}=\frac{-8}{4}$$

$$x=-2$$

Solve $12=\frac{1}{2}(3-x)$ *clear the fraction by multiplying all terms by 2 (careful here)*

$$(2)12=(2)\frac{1}{2}(3-x)$$

$$24=(3-x) \quad \textit{subtract } 3 \textit{ from both sides}$$

$$24-3=3-3-x$$

$$21=-x \quad \textit{divide both sides by } -1$$

$$\frac{21}{-1}=\frac{-x}{-1}$$

$$-21=x$$

$$x=-21$$

Simplify if required, then solve each equation.

85. $x+8=12$

86. $2x-5=17$

87. $5x+8=-42$

88. $4x+2=3x-8$

89. $6x-12=2x$

90. $5(x-3)=3x+1$

91. $\frac{2}{3}(x+4)=8$

92. $\frac{1}{4}(x-2)=3(x-3)$

Problems Involving Algebraic Equations

Example 1:

The sum of two consecutive **even** numbers is 30. Find the numbers. Examples of even numbers are 6, 8, 10, etc.

Examples of odd numbers are 3, 5, 7, etc.

Let the first number = x

Then the second number = $x + 2$ ($+ 2$ because $+ 1$ would make the next number an odd number)

First number + second number = 30

$$\begin{aligned} x + (x+2) &= 30 \\ 2x + 2 &= 30 \\ 2x &= 30 - 2 \\ 2x &= 28 \\ x &= 14 \end{aligned}$$

First number is 14, second number is 16

Example 2:

A father and son have a combined age of 78 years. The father is twice as old as his son. How old is the father and how old is the son?

Son's age $= x$

Father's age $= 2x$

$$\begin{aligned} 2x + x &= 78 \\ 3x &= 78 \\ x &= 26 \end{aligned}$$

The son is 26 years old and the father is 2×26 = 52 years old.

Solve each problem by identifying what x represents, then writing an algebraic equation, and then solving the equation. (Remember to show your algebraic expressions as well as the answer.)

93. Jim ran 2 km less than Steven. They ran a total distance of 12 km. How far did each person run?

    Let Jim's distance =

    Then let Steven's distance =

94. The sum of two numbers is 22. The larger number is four more than the smaller. What are the two numbers?

95. Mary is twice as old as her daughter. The sum of their ages is 66 years. How old is Mary and how old is her daughter?

96. A ribbon is cut into two pieces. One piece is 10 metres longer than the other. How long is each piece if the ribbon was 22 metres long?

97. John weighs 2.5 kg less than Jim. The sum of their weights is 121.5 kg. What does each person weigh?

Manipulating Formulas

Formula manipulation is often useful to rearrange a formula to solve for an unknown variable. To solve a formula for a given variable you must isolate the variable on one side of the equal sign by removing all other terms from the same side.

Remember that formulas are equations. Whatever operations you perform on one side of the equal sign must also be performed on the other side. Use the same principles in solving that you use for any other equations.

Some guidelines to follow when rearranging formulas are:

1. Remove any fractions by multiplying each term by the lowest common denominator.

2. Think about **BEDMAS** in reverse.
   - Move any term added or subtracted first (addition is the opposite of subtraction)
   - Divide by the term in front of or behind the variable you are solving for (division is the opposite of multiplication)
   - If you have a variable squared, then take the $\sqrt{}$ of the variable and the $\sqrt{}$ of the other side of the equal sign. Square rooting ($\sqrt{x}$) is the opposite of squaring (x^2).

Example 1:

$B = EC + D$ is a formula. Solve for C.

Solution

To solve the formula for C, you need to get C alone on one side.

$B = EC + D$	*solve for C*
$B - D = EC + D - D$	*subtract D from each side*
$B - D = EC$	
$\frac{B-D}{E} = \frac{EC}{E}$	*divide both sides by E*
$\frac{B-D}{E} = C$	*written as* $C = \frac{B-D}{E}$

Example 2:

$A=\frac{1}{2}bh$ finds the measure of the area (A) of a triangle with base (b) and height (h). Solve for b.

Solution

To solve for b move A to the right side of the equal sign and $\frac{1}{2}bh$ to the left side of the equal sign to prepare for $b =$

$\frac{1}{2}bh=A$ *multiply both sides by 2 to get rid of the fraction*

$$2\left(\frac{1}{2}bh\right)=2A$$

$bh=2A$ *divide both sides by h*

$$\frac{bh}{h}=\frac{2A}{h}$$

$$b=\frac{2A}{h}$$

Example 3:

$A=\pi r^2$ is the formula for the area (A) of a circle where r is the radius and π is a constant. Solve for r.

Solution

$\pi r^2 = A$ *solve for r*

$\frac{\pi r^2}{\pi}=\frac{A}{\pi}$ *divide both sides by* π

$$r^2=\frac{A}{\pi}$$

$\sqrt{r^2}=\sqrt{\frac{A}{\pi}}$ *take the square root of both sides*

$$r=\sqrt{\frac{A}{\pi}}$$

Solve for the indicated letter.

98. $y = a - x$, for x

99. $y = q - x$, for q

100. $By = Ax$, for y

101. $y = bx + c$, for x

102. $P = \dfrac{a+b}{2}$, for a

103. $P = \dfrac{ab}{c}$, for c

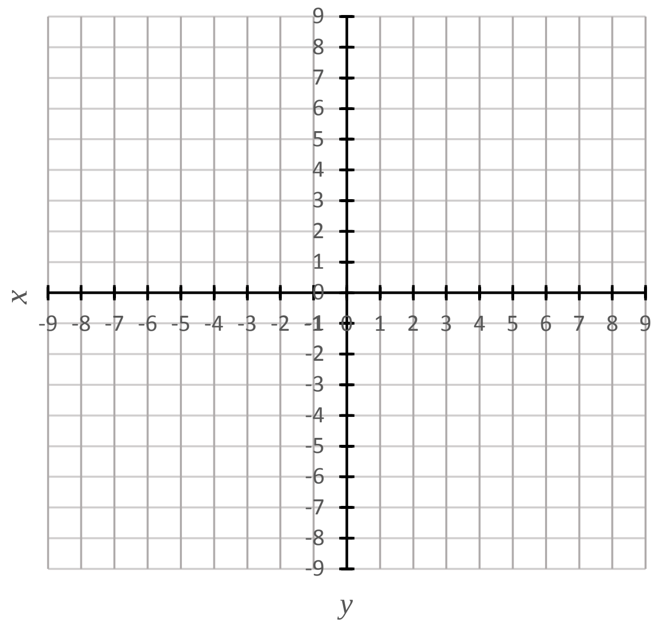
Intermediate Level Graphing

GRAPHING OF LINEAR EQUATIONS

Graph each of the following.

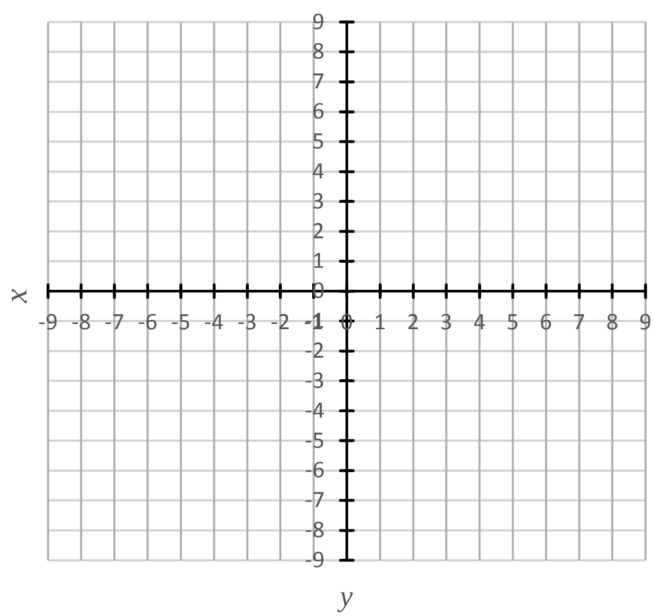
104. $y = x - 4$

x	y



105. $y = 2x + 5$

x	y



Powers and Exponents

REVIEW

A little number placed at the upper right of a number is called a power or exponent. A power is an instruction to multiply the number (or base) by itself, a certain number of times.

number or base $\rightarrow 2^2 \leftarrow$ **power or exponent**

2^3	is read as	two to the third power, or two to the power of three, or two cubed	$2^3 = 2\times 2\times 2 = 8$
5^2	is read as	five to the power of two, or five squared	$5^2 = 5\times 5 = 25$
3^4	is read as	three to the power of four	$3^4 = 3\times 3\times 3\times 3 = 81$
10^5	is read as	ten to the power of five	$10^5 = 10\times 10\times 10\times 10\times 10 = 100,000$
Note that the base of 1 to any power equals 1.			$1^4 = 1\times 1\times 1\times 1 = 1$

The power of 1 is understood for any number but not written for simplicity's sake.

For example:

$8^1 = 8$ and $10^1 = 10$

DECIMALS

Decimals can have powers. These are calculated in the same way as whole numbers.

$0.03^2 = 0.03\times 0.03 = 0.0009$

$2.5^3 = 2.5\times 2.5\times 2.5 = 15.625$

FRACTIONS

To take a fraction to a power, it is necessary to use a bracket around the entire fraction.

$$\left(\frac{2}{3}\right)^3 = \frac{2\times 2\times 2}{3\times 3\times 3} = \frac{8}{27}$$

$$\frac{2^3}{3} = \frac{2\times 2\times 2}{3} = \frac{8}{3}$$

Notice the difference between $\left(\frac{2}{3}\right)^3$ and $\frac{2^3}{3}$

Scientific notation

Very large numbers or very small numbers, often encountered in science, are usually written using a power of the base 10 to make the number more easily and accurately read. Such numbers are said to be written in the scientific notation.

Examples:

decimal notation $\rightarrow$ $265\ 000 = 2.65\times 10^5$ $\leftarrow$ *scientific notation*

decimal notation $\rightarrow$ $0.000\ 000\ 14 = 1.4\times 10^{-7}$ $\leftarrow$ *scientific notation*

Convert each number to decimal notation.

106. 3.2×10^4

107. 6.07×10^7

108. 2.1×10^{-3}

109. 4.5×10^{-5}

Convert each number to scientific notation.

110. 5 320 000

111. 0.005 3

112. 203 000

113. 0.05

Operations with Powers and Square Roots

OPERATIONS WITH POWERS – REVIEW

Remember the following rules:

If you have a **common base number**:

$a^x \times a^y = a^{x+y}$	When multiplying, add the exponents
$\dfrac{a^x}{a^y} = a^{x-y}$	When dividing, subtract the exponents
$(a^x)^y = a^{xy}$	When raising to another power, multiply the exponents
$(a^x b^y)^z = a^{xz} b^{yz}$	When raising several powers to another power, multiply the appropriate exponents

Examples:

$5^4 \times 5^7 = 5^{11}$

$\dfrac{6^3}{6^8} \times 6^{3-8} = 6^{-5} \; or \; \dfrac{1}{6^5}$

$(2^3)^6 = 2^{3\times 6} = 2^{18}$

$(2x^3)^4 = (2^1 x^3)^4 = 2^4 x^{12} \; or \; 16x^{12}$

SQUARE ROOTS – REVIEW

The square root $\left(\sqrt{\;}\right)$ of a number means to find a number which when multiplied by itself is the original number.

As $5 \times 5 = 25$, then the square root of 25 is 5 or $\sqrt{25} = 5$

You should know the following:

$\sqrt{1} = 1$	$\sqrt{4} = 2$	$\sqrt{9} = 3$	$\sqrt{16} = 4$	$\sqrt{25} = 5$	$\sqrt{36} = 6$	$\sqrt{49} = 7$
$\sqrt{64} = 8$	$\sqrt{81} = 9$	$\sqrt{100} = 10$	$\sqrt{121} = 11$	$\sqrt{144} = 12$	$\sqrt{169} = 13$	

Fractions

Notice the difference between:

$\frac{\sqrt{25}}{4} = \frac{5}{4}$ and $\sqrt{\frac{25}{4}} = \frac{5}{2}$

Decimals

Notice that the square root of the decimal 0.09 is 0.3 because:

$0.3 \times 0.3 = 0.09$

Also, $\sqrt{0.04} = 0.2$ $\sqrt{0.0144} = 0.12$ $\sqrt{0.25} = 0.5$

Square Roots that are not Whole Numbers

If we are trying to find the square root of 6, there is no whole number, that when multiplied by itself, is equal to 6. The square root of most numbers is a decimal which goes on forever and doesn't repeat. A calculator will round off these square roots to either 8 or 10 decimal places. You can use a calculator to find these square roots. Always round off the answer to the number of decimal places required.

$\sqrt{10} = 3.16$ (to 2 decimal places) $\sqrt{2} = 1.41$

$\sqrt{91} = 9.54$ $\sqrt{6} = 2.45$

Square Roots Using Variables

As $x^3 \times x^3 = x^6$, then the square root of x^6 is x^3 or $\sqrt{x^6} = x^3$

Consider $\sqrt{x^4}$ as $\sqrt{x \bullet x \bullet x \bullet x}$. Now how many groups of $x \bullet x$ can you make out of $x \bullet x \bullet x \bullet x$?

2 groups, right? Therfore, $\sqrt{x^4} = x^2$

Consider $\sqrt{y^8}$. How many groups of y^2 are there? y^4, right?

What about $\sqrt{\frac{36x^5}{16}}$? That gives $\frac{6x^2}{4}\sqrt{x}$ with $\sqrt{x}$ left over. This equals $\frac{6x^2}{4}\sqrt{x}$

Simplify the following.

114. $x^6 \bullet x^2$

115. $\frac{3^5}{3^2}$

116. $\frac{(2x)^6}{(2x)^5}$

117. $(x^3)^2$

118. $(-3y^2)^3$

119. $\left(\frac{ab}{c}\right)^3$

Find the following.

120. 2^2

121. 5^3

122. 3^3

Evaluate the following.

123. $\sqrt{64}$

124. $\sqrt{121}$

Evaluate the following (round off to 2 decimal places).

125. $\sqrt{13}$

126. $\sqrt{31}$

Find the following.

127. $\sqrt{\frac{25x^8}{9}}$

128. $\sqrt{x^2y^2}$

129. $\sqrt{49y^{10}}$

Intermediate Level Trigonometry

The following trigonometric ratios apply to **right angled** triangles:

$$\text{sin of angle} = \frac{\text{length of side opposite (across from) the angle}}{\text{length of hypotenuse of triangle}} \quad \text{or} \quad s = \frac{o}{h}$$

$$\text{cos of angle} = \frac{\text{length of side adjacent to (beside) angle}}{\text{length of hypotenuse of triangle}} \quad \text{or} \quad c = \frac{a}{h}$$

$$\text{tan of angle} = \frac{\text{length of side opposite the angle}}{\text{length of side adjacent to angle}} \quad \text{or} \quad t = \frac{o}{a}$$

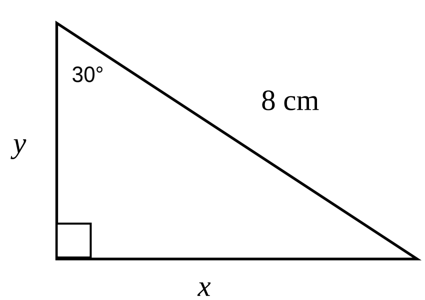
S O H C A H T O A

Using these ratios we can find missing measurements in a triangle. ***You need to use the tables of Trigonometric Ratios or a scientific calculator.***

Examples:

Find the lengths of two sides using a trigonometric ratio.

Find x and y in the triangle using a trigonometric ratio (round to 3 decimal places).



$$\sin 30° = \frac{o}{h}$$

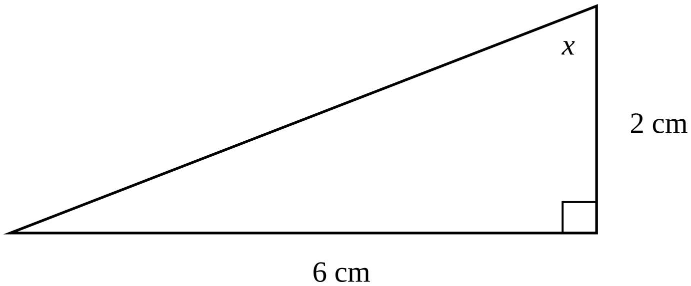
$$\frac{x}{8\text{ cm}} = \sin 30°$$
$$x = \sin 30° \times 8\text{ cm}$$
$$x = 0.5 \times 8\text{ cm}$$
$$x = 4\text{ cm}$$

$$\cos 30° = \frac{a}{h}$$

$$\frac{y}{8\text{ cm}} = \cos 30°$$
$$y = \cos 30° \times 8\text{ cm}$$
$$y = 0.866 \times 8\text{ cm}$$
$$y = 6.928\text{ cm}$$

Find an angle using a trigonometric ratio.

Find the angle marked x in the triangle (round to 1 decimal place).



$$\tan x = \frac{o}{a}$$

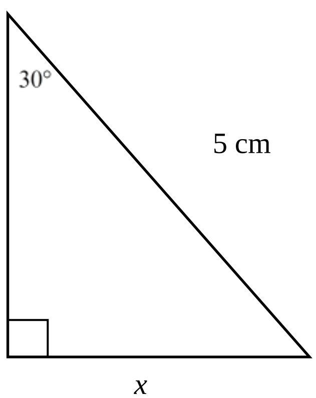
$$\tan x = \frac{6}{2}$$

$$\tan x = 3$$

$$x = 71.6°$$

Use trigonometric ratios or a scientific calculator to find the length of the side or angle indicated in each diagram.

130.



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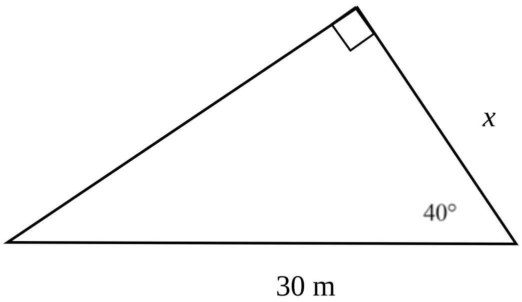

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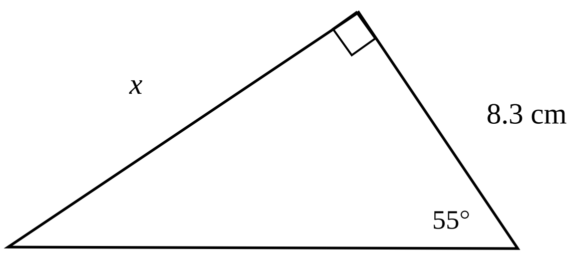

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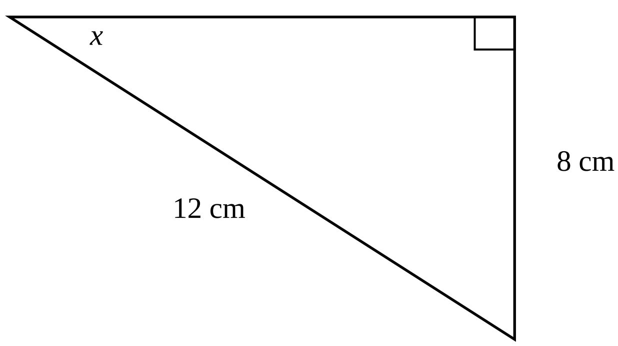


Answers - Part B

Intermediate Level Algebra

1.	25	2.	114	3.	100	4.	0	5.	90
6.	9	7.	$\frac{22}{45}$	8.	28	9.	93	10.	1

Signed Numbers

11.	3	12.	1	13.	-10	14.	-10	15.	32
16.	-4	17.	8	18.	-9	19.	-31	20.	5

Ratio and Proportion

21.	2:5	22.	6:9:14	23.	Yes, 216 = 216	24.	Yes, 15 = 15
25.	$a = \frac{7}{6}\ or\ 1.1\overline{6}$	26.	$d = 6\frac{3}{8}\ or\ 6.375$	27.	3,833 children	28.	CT at 0.0675

Percent

29. a) 0.66 b) 2.357

30. a) $2\frac{11}{20}$ b) $\frac{2}{3}$

31. a) 0.006% b) 675%

32. 50%

33. I = $6,400

Measurement

34. 240,000 m^2

35. 4,320,000 m^2

36. 21.5 cm^2

37. 20.25 m^2 or $20\frac{1}{4}\text{m}^2$

38. 25,500,000,000 m^3

39. 0.000 456 78 m^3

40. 7700 dl^3

41. 64 cm^3

42. 46.277 m^3

43. 54 kg

44. 1,000,000 g

45. 167.64 cm

46. 42.182 km

47. 1,616.24 kg

48. 125,000 cm^3 =125,000 mls = 125 L

Perimeter, Area and Volume

49. $6 \times 8.5 \times 0.25 = 12.75 \text{ m}^3$

50.
$$A = \pi r^2$$
$$706.5 = \pi r^2$$
$$\frac{706.5}{\pi} = r^2$$
$$r^2 = 225 \text{ (rounded)}$$
$$r = 15$$
$$d = 2r = 30 \text{ cm}$$

51.
$$V = \pi r^2 h$$
$$V = 706.5 \times 16$$
$$V = 11{,}304 \text{ cm}^3$$

52.
$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 25$$
$$c = 5 \text{ m}$$

Polynomials

53. 16

54. 17

55. $-7x^3 - x^2 + 3$

56. $6a - 13b + 4c + 2$

57. $5ab^2 - 13ab - 8c$

58. $-2x^2 - 2x + 7$

59. $-6x^4 - 3x^3 - 3x^2 - x$

60. $-2a^2 + 4a$

61. $-3a^3 + 3a^2 - 12a$

62. $4x^2 - 18x + 18$

63. $2x^4 - 4x^3 + 17x^2 - 24x + 30$

64. $24x^3 + 26x^2 - 19x - 6$

65. $6x^3 + 7x^2 - 34x - 35$

66. $4x^2 + 20x + 25$

67. $9x^2 - 6x + 1$

68. $4a^2 - 24a + 36$ *or* $36 - 24a + 4a^2$

69. $-2p^2 + p - 3$

70. $x + y$

71. $r - 3$

72. $\frac{225}{80} = 2.8125 \text{ hrs}$

73. $90 + 0.06(800) = \$138.00$

Factoring

74. $4(2x + 3y)$

75. $2x^2y^2(xy - 4)$

76. $8(x^3 - y - 2)$

77. $(x - 8)(x - 1)$

78. $(r - 8)(r + 2)$

79. $(y + 7)(y - 6)$

80. $(x-4)(x+4)$

81. $2(x-5)(x+5)$

82. $(3x-2)(3x+2)$

83. 480 km

84. 4 km/hr

Solving Equations

85. 4

86. 11

87. -10

88. -10

89. 3

90. 8

91. 8

92. $\frac{34}{11}$ *or* $3\frac{1}{11}$

93. $x+(x-2)=12$ Jim ran 5 km and Steven ran 7 km.

94. $x+(x+4)=22$ The numbers are 9 and 13.

95. $2x+x=66$ Mary is 44 and her daughter is 22 (x is the daughter's age).

96. $x+(x+10)=22$ The pieces of ribbon are 6 m and 16 m long.

97. $(x-2.5)+x=121.5$ John weighs 59.5 kg and Jim weighs 62 kg.

Manipulating Formulas

98. $x=a-y$

99. $q=y+x$

100. $y=\frac{Ax}{B}$

101. $x=\frac{y-c}{b}$

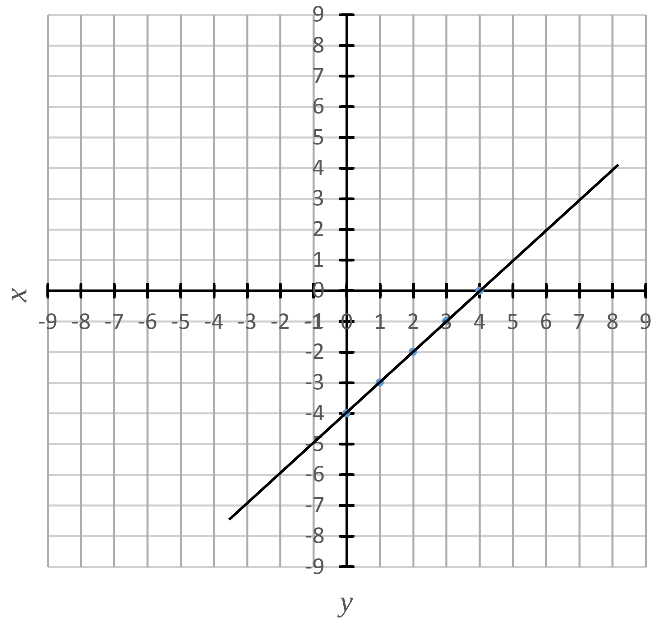
102. $a=2P-b$

103. $c=\frac{ab}{P}$

Graphing Linear Equations

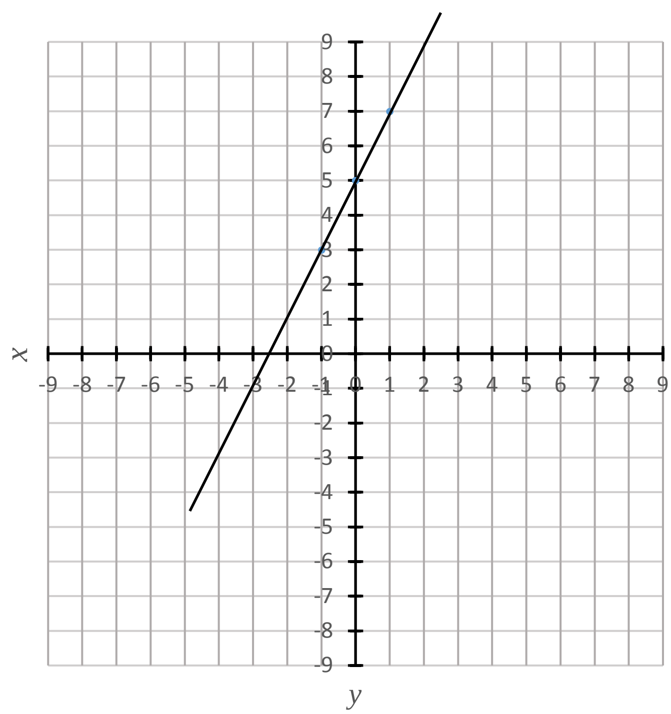
104. $y = x - 4$

x	y
0	-4
1	-3
2	-2
3	-1
4	0



105. $y = 2x + 5$

x	y
-1	3
0	5
1	7



Power or Exponents

106. 32 000
107. 60 700 000
108. 0.002 1
109. 0.000 045
110. 5.32×10^{6}
111. 5.3×10^{-3}
112. 2.03×10^{5}
113. 5.0×10^{-2}

Square Roots

114. x^8
115. 3^3 *or* 27
116. $2x$
117. x^6
118. $(-3)^3 y^6$ *or* $-27y^6$
119. $\dfrac{a^3b^3}{c^3}$
120. 4
121. 125
122. 27
123. 8
124. 11
125. 3.61
126. 5.57
127. $\dfrac{5x^4}{3}$
128. xy
129. $7y^5$

Intermediate Level Trigonometry

130. $\dfrac{x}{5}=\sin 30°$ $\quad x=2.5\text{ cm}$

131. $\dfrac{x}{30}=\cos 40°$ $\quad x=22.98\text{ m}$

132. $\dfrac{x}{8.3}=\tan 55°$ $\quad x=11.85\text{ cm}$ $x=11.85\text{ cm}$

133. $\sin x=\dfrac{8}{12}$ $\quad x=41.8°$

Appendix A: Metric Unit Symbols and Relationships

Time

second	s	
minute	min	1 min = 60 s
hour	h	1 h = 60 min
day	d	1 d = 24 h
year	a	1 a = 365 d

Prefix

mega	M	1000000
kilo	k	1000
hector	h	100
deca	da	10
deci	d	0.1
centi	c	0.01
milli	m	0.001
micro	μ	0.000001

Length

kilometer	km	1 km = 1000 m
metre	m	1 m = 100 cm
centimetre	cm	1 cm = 10 mm
millimetre	mm	1 m = 1000 mm
hectare	ha	1 ha = 1hm^2 = 10 000 m^2

Volume

kiloliter	kl	1 kl = 1000 L
cubic metre	m^3	1 m^3 = 1 kl
litre	L	1 L = 1000 ml
cubic centimetre	cm^3	1 L = 1000 cm^3
milliliter	mL	1 ml = 1 cm^3

Water

1 kl of water = 1 t of water
1 L of water = 1 kg of water
1 ml of water = 1 g of water

Appendix B: Imperial Units

Imperial Units

Length	Liquid Measure	Weight
1 foot (ft.) = 12 inches (in.) 1 yard (yd.) = 3 feet 1 mile (mi.) = 5 280 feet	1 pint (pt.) = 2.5 cups 1 quart (qt.) = 2 pints 1 gallon (gal.) = 4 quarts	1 pound (lb.) = 16 ounces (oz.) 1 ton = 2000 lbs.

Metric / Imperial Conversions

Metric to Imperial	Imperial to Metric
Length 1 cm = 0.394 inches 1 m = 39.4 inches 1 km = 0.621 miles	***Length*** 1 inch = 2.54 cm 1 yard = 0.914 m 1 mile = 1.61 km
Area 1 hectare = 2.47 acres 1 km^2 = 0.386 square miles	***Area*** 1 acre = 0.405 ha 1 sq. mil. = 2.59 km^2
Liquid Measure 1 L = 0.220 gallons 1 L = 4.40 cups	***Liquid Measure*** 1 gallon = 4.55 L 1 cup = 0.228 L
Mass / Weight 1 g = 0.0353 ounces 1 kg = 2.20 pounds	***Mass / Weight*** 1 ounce = 28.4 g 1 pound = 0.454 kg